

VU Research Portal

Non-universality of transverse momentum dependent parton distribution functions

Bomhof, C.J.; Mulders, P.J.G.

published in

Nuclear Physics B
2008

DOI (link to publisher)

[10.1016/j.nuclphysb.2007.11.024](https://doi.org/10.1016/j.nuclphysb.2007.11.024)

document version

Publisher's PDF, also known as Version of record

[Link to publication in VU Research Portal](#)

citation for published version (APA)

Bomhof, C. J., & Mulders, P. J. G. (2008). Non-universality of transverse momentum dependent parton distribution functions. *Nuclear Physics B*, 795(1-2), 409-427. <https://doi.org/10.1016/j.nuclphysb.2007.11.024>

General rights

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal ?

Take down policy

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

E-mail address:

vuresearchportal.ub@vu.nl

Non-universality of transverse momentum dependent parton distribution functions

C.J. Bomhof, P.J. Mulders^{*}

Department of Physics and Astronomy, Vrije Universiteit Amsterdam, NL-1081 HV Amsterdam, The Netherlands

Received 13 September 2007; received in revised form 21 November 2007; accepted 26 November 2007

Available online 4 December 2007

Abstract

In the field theoretical description of hadronic scattering processes, single transverse-spin asymmetries arise due to gluon initial and final state interactions. These interactions lead to process dependent Wilson lines in the operator definitions of transverse momentum dependent parton distribution functions. In particular for hadron–hadron scattering processes with hadronic final states this has important ramifications for possible factorization formulas in terms of (non)universal TMD parton distribution functions. In this paper we will systematically separate the universality-breaking parts of the TMD parton correlators from the universal T -even and T -odd parts. This might play an important role in future factorization studies for these processes. We also show that such factorization theorems will (amongst others) involve the gluonic pole cross sections, which have previously been shown to describe the hard partonic scattering in weighted spin asymmetries.

© 2007 Elsevier B.V. All rights reserved.

PACS: 13.85.Ni; 12.38.Cy; 12.39.St; 11.15.Tk

Keywords: Gauge invariance; Factorization; Partonic intrinsic transverse momentum; Distribution functions; Hard processes; Universality

1. Introduction

Many theoretical as well as experimental studies in recent years have been aimed at better understanding the processes that cause spin asymmetries in hadronic scattering. A mechanism to generate single-spin asymmetries (SSA) through soft gluon interactions between the target

^{*} Corresponding author.

E-mail addresses: cbomhof@nat.vu.nl (C.J. Bomhof), mulders@few.vu.nl (P.J. Mulders).

remnants and the initial and final state partons was first proposed in the context of collinear factorization [1–8]. This collinear factorization formalism involves, apart from the usual twist-two quark correlators, also twist-three collinear quark–gluon matrix elements. Since they contain the field operator of a zero-momentum gluon, they are referred to as *gluonic pole matrix elements*. An important example is the Qiu–Sterman matrix element $T_F(x, x)$ [1–5].

Several other mechanisms to generate SSA's through the effects of the intrinsic transverse momenta of the partons have also been proposed. For instance, in the Siverson effect the asymmetry arises in the initial state due to a correlation between the intrinsic transverse motion of an unpolarized quark and the transverse spin of its parent hadron [9,10]. The effect can be described by a transverse momentum dependent (TMD) distribution function $f_{1T}^\perp(x, p_T^2)$ [11]. Such a function can exist by the grace of soft gluon interactions between the target remnants and the active partons [12,13]. These interactions give rise to process dependent Wilson lines, also called gauge links, in the definitions of TMD parton distribution and fragmentation functions. The Wilson lines secure the gauge invariance of these definitions. At the same time they prevent the use of time-reversal to argue that the Siverson effect should vanish. Instead, time-reversal can be used to derive non-trivial ‘universality’ relations between the Siverson functions in different processes. For instance, it was shown that the Siverson function in SIDIS, which contains a future pointing Wilson line, has opposite sign [12–14] as the TMD function in Drell–Yan scattering, which involves a past pointing Wilson line. Moreover, the Wilson lines are also crucial ingredients in the derivation [15] of the relation between the Siverson function and the Qiu–Sterman matrix element, $2Mf_{1T}^{\perp(1)}(x) = -gT_F(x, x)$, demonstrating that the first transverse moment of the Siverson function is a gluonic pole matrix element.

The process dependence of the Wilson lines in TMD parton correlators makes the study of the (non)universality of these functions particularly important. In the basic electroweak processes, SIDIS, Drell–Yan scattering and e^+e^- -annihilation, the hard partonic parts of the process are just simple electroweak vertices (at tree-level). Depending on the particular process only initial or final-state gluon interactions contribute and, correspondingly, only future and past pointing Wilson lines occur. However, when going to hadronic processes that involve hard parts with more colored external legs, such as in hadronic dijet or photon-jet production, there can be both initial and final state gluon interactions. As a result, the Wilson lines resulting from a resummation of all exchanged collinear gluons will also be more complicated than just the simple future and past pointing Wilson lines [16–18]. In particular, for each of the Feynman diagrams that contribute to the hard partonic part of the hadronic scattering process there is, in principle, a different gauge link structure.

For the TMD distribution functions this at first sight seems to complicate things considerably. However, for the collinear distribution functions remarkable simplifications occur. Upon integration over intrinsic transverse momenta all the effects of the complicated gauge link structures in the TMD correlators disappear, while for the transverse moment they contribute a gluonic pole matrix element with multiplicative prefactors, referred to as *gluonic pole strengths*. These are color-fractions that, in principle, differ for each Feynman diagram that contributes to the partonic subprocess. Therefore, for a given subprocess one can multiply the color factors with the contribution of each partonic diagram and collect them in modified (but manifestly gauge invariant) hard cross sections [17,19]. These modified hard functions, called *gluonic pole cross sections*, appear whenever gluonic pole matrix elements (such as the first moments of the Siverson and Boer–Mulders functions) contribute. This is typically the case in weighted azimuthal spin asymmetries.

The effects of the gluon initial and final-state interactions for the fully TMD treatment of these processes is less clear-cut. In Refs. [20,21] a TMD factorization formula based on one-gluon radiation was proposed for the quark-Sivers contribution to the SSA in dijet production in proton–proton scattering. This result involves the gluonic pole cross sections found in Refs. [17,19] as hard parts, folded with the TMD distribution functions as measured in SIDIS (i.e. with a future pointing Wilson line in their definitions). On the other hand, in Refs. [16–18] it was observed that complicated Wilson line structures occur in the TMD distribution (and fragmentation) functions in such processes. Those results, in concurrence with a model calculation, led the authors of Ref. [22] to conclude that a TMD factorization formula for spin asymmetries in processes such as dijet production in proton–proton scattering cannot be written down with universal distribution functions. It is also asserted that a proof of TMD factorization for such processes will be essentially different from the existing proofs for SIDIS and Drell–Yan scattering and that it will probably involve ‘effective’ TMD parton distribution functions [23]. Recent extensions [24,25] of the work in [20–22] also include the contributions of two collinear gluons (as was previously discussed for Drell–Yan [26]). These indicate that the Feynman graph calculations in Refs. [16–19], [20,21] and [22] are “mutually consistent” up to two-gluon contributions [24].

By using the gluonic pole strengths we will in this paper systematically separate the universality-breaking parts of the TMD parton correlators from the universal T -even and T -odd matrix elements. It is a non-trivial observation that this is possible and we believe that it constitutes another important ingredient in trying to relate the results of Refs. [16–19] and Refs. [20, 21]. We demonstrate that the gluonic pole cross sections are also encountered in unintegrated, unweighted processes. In particular, we will argue that the gluonic pole cross sections can also emerge in unweighted spin-averaged processes and that ordinary partonic cross sections can also arise in unweighted single-spin asymmetries, though they appear in such a way that they will vanish for the integrated and weighted processes [17,19,27,28], respectively. We will start by recapitulating the collinear case in Section 2 and the appearance of universal collinear correlators in hadronic cross sections in Section 3. The study of the non-universality of the TMD parton correlators will be presented in Sections 4 and 5, followed by a discussion on how the non-universal TMD correlators affect hadronic cross sections (Section 6). After summarizing in Section 7 we list all universality-breaking matrix elements that are encountered at tree-level in $2 \rightarrow 2$ hadronic scattering processes (Appendix A).

2. Collinear correlators

For a twist analysis of hadronic variables in high-energy physics it is useful to make a Sudakov decomposition $p^\mu = x P^\mu + \sigma n^\mu + p_T^\mu$ of the momentum p^μ of each active parton. The Sudakov vector n is an arbitrary light-like four-vector $n^2 \equiv 0$ that has non-zero overlap $P \cdot n$ with the hadron’s momentum P^μ . We will choose the Sudakov vector such that this overlap is positive and of the order of the hard scale. Up to subleading twist its coefficient $\sigma = (p \cdot P - x M^2)/(P \cdot n)$ is always integrated over. The vector p_T is called the intrinsic transverse momentum of the parton. It is orthogonal to both P and n , i.e. $p_T \cdot P = p_T \cdot n = 0$, and will appear suppressed by one power of the hard scale with respect to the collinear term. Vectors in the transverse plane can be obtained by using the transverse projectors $g_T^{\mu\nu} \equiv g^{\mu\nu} - P^{(\mu} n^{\nu)}/(P \cdot n)$ and $\epsilon_T^{\mu\nu} \equiv \epsilon^{\mu\nu\rho\sigma} P_\rho n_\sigma/(P \cdot n)$. Note that each observed hadron can have a different transverse plane.

We consider collinear quark distribution functions as being obtained from transverse momentum dependent (TMD) quark distribution functions. Those are projections of the TMD quark

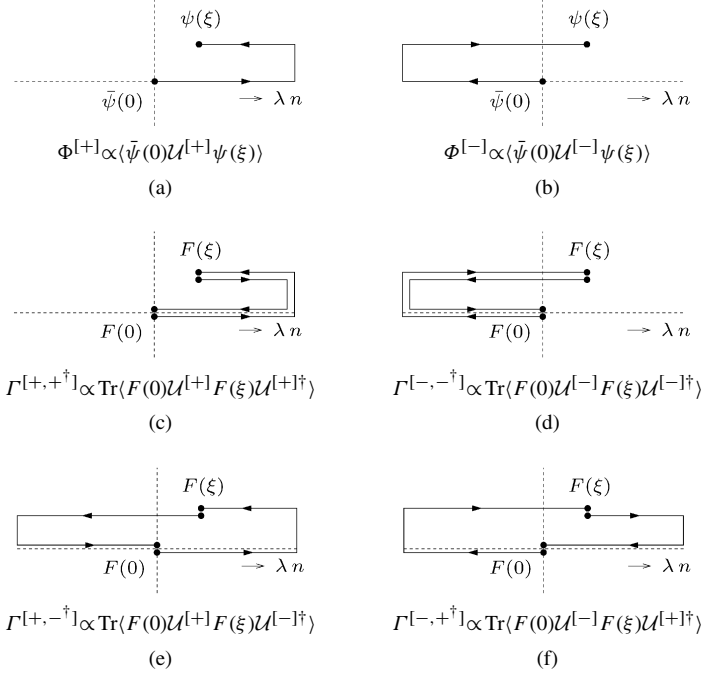


Fig. 1. Simplest structures (without loops) for gauge links and operators in quark correlators (a)–(b) and gluon correlators (c)–(f).

correlator defined on the light-front (LF: $\xi \cdot n \equiv 0$)

$$\Phi_{ij}^{[\mathcal{U}]}(x, p_T; P, S) = \int \frac{d(\xi \cdot P) d^2 \xi_T}{(2\pi)^3} e^{ip \cdot \xi} \langle P, S | \bar{\psi}_j(0) \mathcal{U}_{[0; \xi]} \psi_i(\xi) | P, S \rangle_{\text{LF}}. \quad (1)$$

The *Wilson line* or *gauge link* $\mathcal{U}_{[\eta; \xi]} = \mathcal{P} \exp[-ig \int_C ds \cdot A^a(s) t^a]$ is a path-ordered exponential along the integration path C with endpoints at η and ξ . Its presence in the hadronic matrix element is required by gauge-invariance. In the TMD correlator (1) the integration path C in the gauge link is process-dependent. In the diagrammatic approach the Wilson lines arise by resumming all gluon interactions between the soft and the hard partonic parts of the hadronic process [15,26,29,30]. Consequently, the integration path C is fixed by the (color-flow structure of) the hard partonic scattering [18]. Basic examples are semi-inclusive deep-inelastic scattering (SIDIS) where the resummation of all final-state interactions leads to the future pointing Wilson line $\mathcal{U}^{[+]}$, and Drell–Yan scattering where the initial-state interactions lead to the past pointing Wilson line $\mathcal{U}^{[-]}$, see Figs. 1(a) and (b). All Wilson lines in this text are in the three-dimensional fundamental representation of the color matrices.

Going beyond the simplest electroweak processes such as SIDIS, Drell–Yan scattering and e^+e^- -annihilation, the competing effects of the gluonic initial and final-state interactions lead to gauge link structures that can be quite more complicated than the future or past pointing Wilson lines [16–18]. The situation becomes particularly notorious when considering processes which have several Feynman diagrams that contribute to the partonic scattering. In that case each cut Feynman diagram D can, in principle, lead to a different gauge-invariant correlator

$\Phi^{[\mathcal{U}]}(x, p_T) = \Phi^{[\mathcal{U}(D)]}(x, p_T) \equiv \Phi^{[D]}(x, p_T)$ [16–18]. This observation leads to a broad spectrum of different TMD parton correlators that appear in hadronic scattering processes.

For collinear correlators the situation is simpler. For instance, in the p_T -integrated correlator defined on the lightcone (LC: $\xi \cdot n = \xi_T \equiv 0$)

$$\begin{aligned} \Phi_{ij}(x) &= \int d^2 p_T \Phi_{ij}^{[\mathcal{U}]}(x, p_T) \\ &= \int \frac{d(\xi \cdot P)}{2\pi} e^{ip \cdot \xi} \langle P, S | \bar{\psi}_j(0) U_{[0;\xi]}^n \psi_i(\xi) | P, S \rangle_{\text{LC}}, \end{aligned} \quad (2)$$

all process-dependence of the gauge link disappears, leaving just a straight Wilson line $U_{[0;\xi]}^n$ in the lightcone n -direction, where n is the lightlike vector in the Sudakov decomposition of the quark momentum p (we will use the non-calligraphed letter U to indicate straight line segments). Another situation is encountered in the transverse momentum weighted correlators (the transverse moments). In the transverse moments a (sub)process-dependence remains as a direct consequence of the presence of the gauge links in the TMD correlators. Nevertheless, a simple decomposition can still be made (omitting the Dirac indices) [15,17,19]:

$$\Phi_{\theta}^{[\mathcal{U}] \alpha}(x) \equiv \int d^2 p_T p_T^{\alpha} \Phi^{[\mathcal{U}]}(x, p_T) = \tilde{\Phi}_{\theta}^{\alpha}(x) + C_G^{[\mathcal{U}]} \pi \Phi_G^{\alpha}(x, x), \quad (3)$$

with collinear correlators

$$\Phi_D^{\alpha}(x) = \int \frac{d(\xi \cdot P)}{2\pi} e^{ix(\xi \cdot P)} \langle P, S | \bar{\psi}(0) U_{[0;\xi]}^n i D^{\alpha}(\xi) \psi(\xi) | P, S \rangle_{\text{LC}}, \quad (4a)$$

$$\begin{aligned} \Phi_G^{\alpha}(x, x - x') &= \frac{n_{\mu}}{P \cdot n} \int \frac{d(\xi \cdot P)}{2\pi} \frac{d(\eta \cdot P)}{2\pi} e^{ix'(\eta \cdot P)} \\ &\times e^{i(x-x')(\xi \cdot P)} \langle P, S | \bar{\psi}(0) U_{[0;\eta]}^n g F^{\mu\alpha}(\eta) U_{[\eta;\xi]}^n \psi(\xi) | P, S \rangle_{\text{LC}}, \end{aligned} \quad (4b)$$

and

$$\tilde{\Phi}_{\theta}^{\alpha}(x) = \Phi_D^{\alpha}(x) - \int dx' P \frac{i}{x'} \Phi_G^{\alpha}(x, x-x'). \quad (5)$$

The only process dependence due to the Wilson lines in the TMD correlators resides in the multiplicative factors $C_G^{[\mathcal{U}]} = C_G^{[\mathcal{U}(D)]} \equiv C_G^{[D]}$. They are color-fractions that are fixed by the color-flow structure of the hard partonic function of the scattering process [17,19]. We will refer to them as *gluonic pole strengths*. Important examples are the transverse moments of the correlators $\Phi^{[+]}$ in SIDIS and $\Phi^{[-]}$ in Drell–Yan scattering, for which one has $C_G^{[\pm]} \equiv C_G^{[\mathcal{U}^{[\pm]}]} = \pm 1$ [15].

Transverse momentum dependent gluon distribution functions are projections of the TMD correlator

$$\begin{aligned} \Gamma^{[\mathcal{U}, \mathcal{U}'] \mu\nu}(x, p_T; P, S) \\ = \frac{n_{\rho} n_{\sigma}}{(p \cdot n)^2} \int \frac{d(\xi \cdot P) d^2 \xi_T}{(2\pi)^3} e^{ip \cdot \xi} \text{Tr} \langle P, S | F^{\mu\rho}(0) \mathcal{U}_{[0;\xi]} F^{\nu\sigma}(\xi) \mathcal{U}'_{[\xi;0]} | P, S \rangle_{\text{LF}}. \end{aligned} \quad (6)$$

Here Tr indicates a trace over color-triplet indices. Writing the field-operators in the color-triplet representation requires the inclusion of *two* Wilson lines $\mathcal{U}_{[0;\xi]}$ and $\mathcal{U}'_{[\xi;0]}$ [18]. They again arise from the resummation of gluon initial and final-state interactions. In general this will lead to two unrelated Wilson lines \mathcal{U} and \mathcal{U}' . In the particular case that $\mathcal{U}' = \mathcal{U}^{\dagger}$, the gluon correlator can also be written as the product $\langle F_a \mathcal{U}_{ab} F_b \rangle$ of two gluon fields with the Wilson line \mathcal{U} in the adjoint

representation of $SU(N)$. This is for instance the case for the gluon correlators in Figs. 1(c) and (d), but not for the gluon correlators in Figs. 1(e) and (f).

In the p_T -integrated correlator on the lightcone the process dependence of the TMD gluon correlator disappears,

$$\begin{aligned}\Gamma^{\mu\nu}(x) &= \int d^2 p_T \Gamma^{[\mathcal{U}, \mathcal{U}']^{\mu\nu}}(x, p_T) \\ &= \frac{n_\rho n_\sigma}{(p \cdot n)^2} \int \frac{d(\xi \cdot P)}{2\pi} e^{ix(\xi \cdot P)} \text{Tr}\langle P, S | F^{\mu\rho}(0) U_{[0;\xi]}^n F^{\nu\sigma}(\xi) U_{[\xi;0]}^n | P, S \rangle \rangle_{\text{LC}}.\end{aligned}\quad (7)$$

However, as for the quark correlator, a subprocess-dependence due to the Wilson lines in the TMD gluon correlators remains in the transverse moments. The analogue of the decomposition (3) in the case of the gluon correlator is (with $\Gamma^{[\mathcal{U}, \mathcal{U}']}(x, p_T) = \Gamma^{[\mathcal{U}(D), \mathcal{U}'(D)]}(x, p_T) \equiv \Gamma^{[D]}(x, p_T)$) and omitting the gluon field indices μ and ν) [19]:

$$\Gamma_\partial^{[D]\alpha}(x) = \tilde{\Gamma}_\partial^\alpha(x) + C_G^{(f)[D]} \pi \Gamma_{G_f}^\alpha(x, x) + C_G^{(d)[D]} \pi \Gamma_{G_d}^\alpha(x, x). \quad (8)$$

The matrix elements Γ_{G_f} and Γ_{G_d} are the two gluonic pole matrix elements that correspond to the two possible ways to construct color-singlets from three gluon fields [19,31]. They involve the antisymmetric f and symmetric d structure constants of $SU(3)$, respectively. The only process dependence coming from the Wilson lines in the TMD correlators is contained in the gluonic pole strengths $C_G^{(f/d)[D]} \equiv C_G^{(f/d)[\mathcal{U}(D), \mathcal{U}'(D)]}$. The collinear correlators are

$$\begin{aligned}\Gamma_D^{\mu\nu;\alpha}(x) &= \frac{n_\rho n_\sigma}{(p \cdot n)^2} \int \frac{d(\xi \cdot P)}{2\pi} e^{ix(\xi \cdot P)} \\ &\quad \times \text{Tr}\langle P, S | F^{\mu\rho}(0) U_{[0;\xi]}^n [i D^\alpha(\xi), F^{\nu\sigma}(\xi)] U_{[\xi;0]}^n | P, S \rangle \rangle_{\text{LC}},\end{aligned}\quad (9a)$$

$$\begin{aligned}\Gamma_{G_f}^{\mu\nu;\alpha}(x, x-x') &= \frac{n_\rho n_\sigma}{(p \cdot n)^2} \int \frac{d(\xi \cdot P)}{2\pi} \frac{d(\eta \cdot P)}{2\pi} e^{ix'(\eta \cdot P)} e^{i(x-x')(\xi \cdot P)} \\ &\quad \times \text{Tr}\langle P, S | F^{\mu\rho}(0) [U_{[0,\eta]}^n g F^{n\alpha}(\eta) U_{[\eta,0]}^n, U_{[0,\xi]}^n F^{\nu\sigma}(\xi) U_{[\xi,0]}^n] | P, S \rangle \rangle_{\text{LC}},\end{aligned}\quad (9b)$$

$$\begin{aligned}\Gamma_{G_d}^{\mu\nu;\alpha}(x, x-x') &= \frac{n_\rho n_\sigma}{(p \cdot n)^2} \int \frac{d(\xi \cdot P)}{2\pi} \frac{d(\eta \cdot P)}{2\pi} e^{ix'(\eta \cdot P)} e^{i(x-x')(\xi \cdot P)} \\ &\quad \times \text{Tr}\langle P, S | F^{\mu\rho}(0) \{ U_{[0,\eta]}^n g F^{n\alpha}(\eta) U_{[\eta,0]}^n, U_{[0,\xi]}^n F^{\nu\sigma}(\xi) U_{[\xi,0]}^n \} | P, S \rangle \rangle_{\text{LC}},\end{aligned}\quad (9c)$$

and

$$\tilde{\Gamma}_\partial^\alpha(x) = \Gamma_D^\alpha(x) - \int dx' P \frac{i}{x'} \Gamma_{G_f}^\alpha(x, x-x'). \quad (10)$$

The collinear (anti)quark and gluon fragmentation correlators can be analyzed in the same way. The matrix elements in (3) and (8) contain the collinear T -even and T -odd parton distribution functions, see e.g. [19].

3. Collinear functions in hadronic cross sections

In the diagrammatic approach, the calculation of the hadronic cross sections starts off with the transverse momentum dependent parton correlators (TMD distribution and fragmentation functions), which will appear in combination with squares of hard partonic amplitudes. In general the

hard amplitude contains more terms, that is $H = \sum_i H_i$. In the squared amplitude one therefore has terms like $H_i^* H_j^* \equiv \hat{\Sigma}^{[D]}$, where D refers to the cut Feynman diagram that is the pictorial representation of the product of the amplitude H_j and conjugate hard amplitude H_i^* . The hadronic cross section $d\sigma$ of a hadronic scattering process mediated by a two-to-two partonic subprocess $a(p_1)b(p_2) \rightarrow c(k_1)d(k_2)$ and where the outgoing hadrons and/or jets are produced with large perpendicular component with respect to the beam will contain the following structure in the integrand:

$$\begin{aligned} \Sigma(p_1, p_2, k_1, k_2) \equiv & \sum_{a, \dots, d} \sum_D \Phi_a^{[D]}(x_1, p_{1T}) \otimes \Phi_b^{[D]}(x_2, p_{2T}) \otimes \hat{\Sigma}^{[D]}(p_1, p_2, k_1, k_2) \\ & \otimes \Delta_c^{[D]}(z_1, k_{1T}) \otimes \Delta_d^{[D]}(z_2, k_{2T}), \end{aligned} \quad (11)$$

where the parton momenta are approximately (compared to the hard scale) on-shell. The convolutions ‘ \otimes ’ represent the appropriate Dirac and color traces for the hard function $\hat{\Sigma}^{[D]}$. To get to the hadronic cross section one has to multiply by the flux factor and integrate over the final-state phase-space and parton momenta including a delta function for momentum conservation on the partonic level. Since our aim in this paper is to display some general features that a k_T -factorization formula (if it exists) will have due to the process-dependence of the Wilson lines that arise in the diagrammatic TMD gauge link approach, we focus our discussion on the Wilson lines and neglect soft factors. In the full k_T -factorization formula such factors will most likely also be present to account for soft-gluon effects.

The TMD correlators in (11) are the gauge invariant non-universal (anti)quark/gluon correlators that contain the appropriate Wilson lines for the particular color-flow diagram D that contributes to the partonic subprocess $ab \rightarrow cd$. This is the reason why in expression (11) the summation over cut diagrams D is displayed explicitly. The hard functions, i.e. the expressions $\hat{\Sigma}^{[D]}$ of the individual Feynman diagrams are, themselves, not gauge invariant. For azimuthal dependence originating from only one of the partons one can effectively use the correlators calculated in Ref. [18]. Furthermore, in the tree-level discussion employed here the Wilson lines are along the lightlike n -direction, though a non-lightlike $n^2 \neq 0$ direction may be required when higher-order corrections are taken into account [32,33].

From momentum conservation on the partonic level it will follow that, depending on the process, some components of the partonic momenta can be measured (e.g. in a way similar to the identification of the incoming parton momentum fraction x with the Bjorken scaling variable x_B in deep inelastic scattering). This also works for the transverse momenta. For instance, for a hadronic scattering process with a two-to-two hard subprocess the structure in (11) will appear with a delta function for momentum conservation enforcing the relation $p_1 + p_2 - k_1 - k_2 \equiv 0$. There are ways to measure one or several components of $q_T \equiv p_{1T} + p_{2T} - k_{1T} - k_{2T} \approx K_1/z_1 + K_2/z_2 - x_1 P_1 - x_2 P_2$, which is not required to vanish by momentum conservation since the directions of the intrinsic transverse momenta of the partons can be different for each observed hadron (in back-to-back jet production in hadron–hadron scattering it is the component along the outgoing jet direction in the plane perpendicular to the beam axis that is experimentally accessible through the relation $q_T \cdot \hat{K}_{\text{jet}}^\perp \propto \sin(\delta\phi)$, where $\delta\phi$ is the azimuthal imbalance of the two jets in the perpendicular plane [17,19,27,34]). This quantity defines a scale much smaller than the hard scale of the process. Using these components one can construct integrated and weighted hadronic cross sections. Integrated cross sections will involve the structure

$$\begin{aligned}\Sigma(x_1, x_2, z_1, z_2) &= \int d^2 p_{1T} d^2 p_{2T} d^2 k_{1T} d^2 k_{2T} \Sigma(p_1, p_2, k_1, k_2) \\ &= \sum_{a, \dots, d} \text{Tr} \{ \Phi_a(x_1) \Phi_b(x_2) \hat{\Sigma}_{ab \rightarrow cd}(x_1, x_2, z_1, z_2) \Delta_c(z_1) \Delta_d(z_2) \},\end{aligned}\quad (12)$$

while weighted cross sections will involve (making use of the decomposition in Eq. (3))

$$\begin{aligned}\Sigma_{1\partial}^\alpha(x_1, x_2, z_1, z_2) &= \int d^2 p_{1T} d^2 p_{2T} d^2 k_{1T} d^2 k_{2T} p_{1T}^\alpha \Sigma(p_1, p_2, k_1, k_2) \\ &= \sum_{a, \dots, d} \sum_D \text{Tr} \{ \Phi_{a\partial}^{[D]\alpha}(x_1) \Phi_b(x_2) \hat{\Sigma}^{[D]}(x_1, x_2, z_1, z_2) \Delta_c(z_1) \Delta_d(z_2) \} \\ &= \sum_{a, \dots, d} [\text{Tr} \{ \tilde{\Phi}_{a\partial}^\alpha(x_1) \Phi_b(x_2) \hat{\Sigma}_{ab \rightarrow cd}(x_1, x_2, z_1, z_2) \Delta_c(z_1) \Delta_d(z_2) \} \\ &\quad + \text{Tr} \{ \pi \Phi_{aG}^\alpha(x_1, x_1) \Phi_b(x_2) \hat{\Sigma}_{[a]b \rightarrow cd}(x_1, x_2, z_1, z_2) \Delta_c(z_1) \Delta_d(z_2) \}],\end{aligned}\quad (13)$$

and similar expressions $\Sigma_{2\partial}$, $\Sigma_{1'\partial}$ and $\Sigma_{2'\partial}$ which are obtained by weighting with p_{2T} , k_{1T} and k_{2T} , respectively. In these expressions only universal collinear correlators appear. In Eqs. (12) and (13) we have defined the hard functions

$$\hat{\Sigma}_{ab \rightarrow cd}(x_1, x_2, z_1, z_2) = \sum_D \hat{\Sigma}^{[D]}(x_1, x_2, z_1, z_2), \quad (14a)$$

$$\hat{\Sigma}_{[a]b \rightarrow cd}(x_1, x_2, z_1, z_2) = \sum_D C_G^{[D]}(a) \hat{\Sigma}^{[D]}(x_1, x_2, z_1, z_2). \quad (14b)$$

The factors $C_G^{[D]}(a)$ are the gluonic pole strengths that appear in the decomposition of the transverse moment of the TMD correlator of parton a . In expression (13) this parton was implicitly taken to be a quark. If it were a gluon there would have been two $\hat{\Sigma}_{[g]b \rightarrow cd}$ terms, one corresponding to each of the gluonic pole matrix elements Γ_{G_f} and Γ_{G_d} , cf. Eq. (8) or Ref. [19]. The hard functions in Eqs. (14a) and (14b) no longer depend on the individual (diagrammatic) contributions D , but only on the hard process $ab \rightarrow cd$. Moreover, in contrast to the hard functions $\hat{\Sigma}^{[D]}$ that appear in (11), they are gauge invariant expressions. After performing the traces the $\hat{\Sigma}_{ab \rightarrow cd}$ reduce to the partonic cross sections $d\hat{\sigma}_{ab \rightarrow cd}$ and the $\hat{\Sigma}_{[a]b \rightarrow cd}$ reduce to the gluonic pole cross sections $d\hat{\sigma}_{[a]b \rightarrow cd}$ calculated in Refs. [17,19,28].

4. Transverse momentum dependent correlators

To study azimuthal asymmetries arising from one of the partons in hadronic processes mediated by $2 \rightarrow 2$ partonic subprocesses at tree-level it is possible, as we will show explicitly, to organize the TMD correlators in a decomposition analogous to (3) containing TMD correlators with special properties:

$$\Phi^{[D]}(x, p_T) = \Phi^{(ab \rightarrow cd)}(x, p_T) + C_G^{[D]} \pi \Phi_G^{(ab \rightarrow cd)}(x, p_T). \quad (15)$$

Here D refers to a particular cut Feynman diagram that contributes to the cross section of the partonic process $ab \rightarrow cd$. The gluonic pole factors $C_G^{[D]}$ are the same as those in the decomposition of the collinear correlators in (3). An important difference between the decomposition of the collinear correlator in (3) and the decomposition of the TMD correlator in (15) is that the matrix elements in the latter decomposition are not universal in general. They depend on

the partonic process $ab \rightarrow cd$ but, in contrast to the TMD quark correlators $\Phi^{[D]}$ on the l.h.s. of the decomposition, they do not depend on the individual cut Feynman diagram D . The only diagram dependence resides in the gluonic pole factors. The matrix elements on the r.h.s. of (15) have been chosen such that they reduce to the familiar universal (process independent) collinear matrix elements when integrating over or weighting with the intrinsic transverse momenta:

$$\int d^2 p_T \Phi^{(ab \rightarrow cd)}(x, p_T) = \Phi(x), \quad \int d^2 p_T p_T^\alpha \Phi^{(ab \rightarrow cd)}(x, p_T) = \tilde{\Phi}_\alpha^\alpha(x), \quad (16a)$$

$$\int d^2 p_T \Phi_G^{(ab \rightarrow cd)}(x, p_T) = 0, \quad \int d^2 p_T p_T^\alpha \Phi_G^{(ab \rightarrow cd)}(x, p_T) = \Phi_G^\alpha(x, x). \quad (16b)$$

The most straightforward illustration of the decomposition (15) for quark TMDs are the quark correlators $\Phi^{[+]}(x, p_T)$ in SIDIS and $\Phi^{[-]}(x, p_T)$ in Drell–Yan scattering, which contain the simple future and past-pointing Wilson lines $\mathcal{U}^{[+]}$ and $\mathcal{U}^{[-]}$, respectively. For those correlators one has [15]

$$\Phi^{[\pm]}(x, p_T) = \Phi^{(T\text{-even})}(x, p_T) + C_G^{[\pm]} \Phi^{(T\text{-odd})}(x, p_T), \quad (17)$$

with the T -even and T -odd quark correlators

$$\Phi^{(T\text{-even})}(x, p_T) = \frac{1}{2}(\Phi^{[+]}(x, p_T) + \Phi^{[-]}(x, p_T)), \quad (18a)$$

$$\Phi^{(T\text{-odd})}(x, p_T) = \frac{1}{2}(\Phi^{[+]}(x, p_T) - \Phi^{[-]}(x, p_T)). \quad (18b)$$

The factors $C_G^{[\pm]} = \pm 1$ are the same as those for the transverse moments (3) in those processes.

In contrast to (17), we observe that the TMD matrix elements $\Phi^{(ab \rightarrow cd)}(x, p_T)$ and $\pi \Phi_G^{(ab \rightarrow cd)}(x, p_T)$ on the r.h.s. of (15) in general do not have definite behavior under time-reversal. However, the process dependent universality-breaking parts of the TMD correlators can be separated from the universal T -even and T -odd parts in (18a) and (18b):

$$\Phi^{(ab \rightarrow cd)}(x, p_T) = \Phi^{(T\text{-even})}(x, p_T) + \delta \Phi^{(ab \rightarrow cd)}(x, p_T), \quad (19a)$$

$$\pi \Phi_G^{(ab \rightarrow cd)}(x, p_T) = \Phi^{(T\text{-odd})}(x, p_T) + \pi \delta \Phi_G^{(ab \rightarrow cd)}(x, p_T). \quad (19b)$$

In these expressions all process dependence due to Wilson lines on the light-front is now contained in process-dependent universality-breaking matrix elements $\delta \Phi^{(ab \rightarrow cd)}(x, p_T)$ and $\pi \delta \Phi_G^{(ab \rightarrow cd)}(x, p_T)$, which we will refer to as *junk-TMD*. Also these in general have no definite behavior under time-reversal, but they do have the special properties that they vanish after p_T -integration and weighting:

$$\int d^2 p_T \delta \Phi^{(ab \rightarrow cd)}(x, p_T) = \int d^2 p_T p_T^\alpha \delta \Phi^{(ab \rightarrow cd)}(x, p_T) = 0, \quad (20a)$$

$$\int d^2 p_T \delta \Phi_G^{(ab \rightarrow cd)}(x, p_T) = \int d^2 p_T p_T^\alpha \delta \Phi_G^{(ab \rightarrow cd)}(x, p_T) = 0. \quad (20b)$$

This is consistent with the properties expressed in (16). The expressions in Eq. (20) will be used in Section 6 to show that the universality-breaking matrix elements vanish in integrated and weighted hadronic cross sections. Moreover, as can be seen from their explicit expressions in Appendix A the (anti)quark/gluon universality-breaking matrix elements vanish in an order g expansion of the Wilson lines (i.e. the one-gluon radiation contribution).

All universality-breaking matrix elements that occur at tree-level in proton–proton scattering with hadronic final states are listed in [Appendix A](#). The TMD correlators $\Phi^{[D]}(x, p_T)$ in these processes have already been derived in Ref. [18] and are given in the tables of that reference (the TMD correlators and gluonic pole factors that appear at tree-level in direct photon-jet production in proton–proton scattering can be found in Ref. [35]). It is straightforward to verify that these results are reproduced with the matrix elements given in [Appendix A](#) and through the decompositions (15) and (19). This should not come as a surprise, since the matrix elements in the appendix were defined that way. It is a remarkable and non-trivial observation that with the gluonic pole strengths all the quark correlators encountered in a certain partonic process $ab \rightarrow cd$ can be decomposed in terms of only the two matrix elements $\Phi^{(ab \rightarrow cd)}$ and $\pi \Phi_G^{(ab \rightarrow cd)}$ (with the properties in Eq. (16)). It should be mentioned, though, that this decomposition is not unique. For instance, one could also have made a decomposition in terms of more matrix elements. That is, by including matrix elements that do not contribute to the zeroth (p_T -integration) and first (p_T -weighting) transverse moments in p_T , but do contribute to the second moment, third moment, etc. It is conceivable that the inclusion of these additional matrix elements will allow one to summarize the TMD quark correlators encountered in different partonic processes, in the same way as it was possible to summarize all quark correlators associated to the different Feynman diagrams D that contribute to one specific partonic process $ab \rightarrow cd$ by the two matrix elements $\Phi^{(ab \rightarrow cd)}$ and $\pi \Phi_G^{(ab \rightarrow cd)}$. At present this is just speculation, though, and the verification or falsification will require more insight into the way that the different Wilson line structures contribute to higher transverse moments. However, regardless of all these cautionary remarks we believe that the notational advantage, the points concerning gauge invariance of the hard functions that will be addressed in Section 6 and the possible role that it could play in relating the gauge link formalism to the results of Refs. [20,21] provide more than enough justification for the decomposition in Eq. (15).

In the case of gluon distributions we start by defining T -even and T -odd gluon correlators (cf. [Figs. 1\(c\)–\(f\)](#)),

$$\Gamma^{(T\text{-even})}(x, p_T) = \frac{1}{2}(\Gamma^{[+, +^\dagger]}(x, p_T) + \Gamma^{[-, -^\dagger]}(x, p_T)), \quad (21a)$$

$$\Gamma_{(f)}^{(T\text{-odd})}(x, p_T) = \frac{1}{2}(\Gamma^{[+, +^\dagger]}(x, p_T) - \Gamma^{[-, -^\dagger]}(x, p_T)), \quad (21b)$$

$$\Gamma_{(d)}^{(T\text{-odd})}(x, p_T) = \frac{1}{2}(\Gamma^{[+, -^\dagger]}(x, p_T) - \Gamma^{[-, +^\dagger]}(x, p_T)), \quad (21c)$$

where as in Ref. [15] a gluon correlator is called T -odd if it vanishes when identifying the future and past-pointing Wilson lines. Note that in contrast to $\Gamma^{(T\text{-even})}$ and $\Gamma_{(f)}^{(T\text{-odd})}$, the correlator $\Gamma_{(d)}^{(T\text{-odd})}$ cannot be written as a matrix element of two gluon fields with a single Wilson line in the adjoint representation. After p_T -integration the T -even and T -odd correlators reduce to the universal collinear gluon matrix elements in the expressions in Eqs. (7) and (8):

$$\int d^2 p_T \Gamma^{(T\text{-even})}(x, p_T) = \Gamma(x), \quad \int d^2 p_T p_T^\alpha \Gamma^{(T\text{-even})}(x, p_T) = \tilde{\Gamma}_g^\alpha(x), \quad (22a)$$

$$\int d^2 p_T \Gamma_{(f)}^{(T\text{-odd})}(x, p_T) = 0, \quad \int d^2 p_T p_T^\alpha \Gamma_{(f)}^{(T\text{-odd})}(x, p_T) = \pi \Gamma_{G_f}^\alpha(x, x), \quad (22b)$$

$$\int d^2 p_T \Gamma_{(d)}^{(T\text{-odd})}(x, p_T) = 0, \quad \int d^2 p_T p_T^\alpha \Gamma_{(d)}^{(T\text{-odd})}(x, p_T) = \pi \Gamma_{G_d}^\alpha(x, x). \quad (22c)$$

Since there are two distinct ways to construct T -odd gluon correlators, it will follow in the next section that there are also two distinct TMD gluon-Sivers distribution functions (cf. Eq. (28b)). There is actually also a second way to construct a T -even gluon correlator: $\Gamma'^{(T\text{-even})} = \frac{1}{2}(\Gamma^{[+, -^\dagger]} + \Gamma^{[-, +^\dagger]})$. However, this correlator is not needed in the decomposition (23) of the TMD gluon correlators, since the difference between $\Gamma'^{(T\text{-even})}$ and $\Gamma^{(T\text{-even})}$ is a matrix element that vanishes upon p_T -integration and p_T -weighting. This difference may, therefore, be absorbed in the universality-breaking matrix elements $\delta\Gamma^{(ab\rightarrow cd)}$ to be defined in (24a).

A decomposition resembling the one in (8) for TMD gluon correlators can almost be made:

$$\Gamma^{[D]}(x, p_T) = \Gamma^{(ab\rightarrow cd)}(x, p_T) + C_G^{(f)[D]} \Gamma_{G_f}^{(ab\rightarrow cd)}(x, p_T) + C_G^{(d)[D]} \Gamma_{G_d}^{(ab\rightarrow cd)}(x, p_T). \quad (23)$$

This leaves only a specific type of matrix elements with colorless intermediate states unaccounted for (we will return to this point in a moment). The TMD matrix elements on the r.h.s. of (23) only depend on the process and not on the particular Feynman diagram D that contributes to that process. The multiplicative factors $C_G^{(f)}$ and $C_G^{(d)}$ are the gluonic pole factors calculated in Ref. [19], the same that also appear in the decomposition (8) of the collinear correlator. Under p_T -integration and weighting the matrix elements $\Gamma^{(ab\rightarrow cd)}(x, p_T)$ and $\Gamma_{G_{f/d}}^{(ab\rightarrow cd)}(x, p_T)$ have the same behavior as $\Gamma^{(T\text{-even})}(x, p_T)$ and $\Gamma_{(f/d)}^{(T\text{-odd})}(x, p_T)$, respectively. Therefore, one can make the further separation

$$\Gamma^{(ab\rightarrow cd)}(x, p_T) = \Gamma^{(T\text{-even})}(x, p_T) + \delta\Gamma^{(ab\rightarrow cd)}(x, p_T), \quad (24a)$$

$$\pi \Gamma_{G_{f/d}}^{(ab\rightarrow cd)}(x, p_T) = \Gamma_{(f/d)}^{(T\text{-odd})}(x, p_T) + \pi \delta\Gamma_{G_{f/d}}^{(ab\rightarrow cd)}(x, p_T), \quad (24b)$$

in which all process-dependence due to Wilson lines on the light-front has been gathered in the universality-breaking matrix elements $\delta\Gamma^{(ab\rightarrow cd)}(x, p_T)$ and $\pi \delta\Gamma_{(f/d)}^{(ab\rightarrow cd)}(x, p_T)$, which have the special properties that they vanish after a p_T -integration and p_T -weighting.

It is also straightforward to check that through the decompositions in (23)–(24) and with the universality-breaking matrix elements given in Appendix A the TMD gluon correlators in Tables 4, 5 and 8 of Ref. [18] corresponding to $qg \rightarrow qg$, $\bar{q}g \rightarrow \bar{q}g$ and $gg \rightarrow gg$ scattering are reproduced. However, in the TMD correlators in Tables 6 and 7 for the processes $q\bar{q} \rightarrow gg$ and $gg \rightarrow q\bar{q}$ one does not recover terms of the form $\langle P, S | \text{Tr}[F(\xi) \mathcal{U}^{[\square]}] \text{Tr}[F(0) \mathcal{U}^{[\square]^\dagger}] | P, S \rangle$, where $\mathcal{U}^{[\square]} = \mathcal{U}^{[+]} \mathcal{U}^{[-]^\dagger}$. These matrix elements involve colorless intermediate states and they do not contribute to the p_T -integrated gluon correlators $\Gamma(x)$ nor to the first transverse moments $\Gamma_\phi^{[D]}(x)$. They can be included in (23) by adding diagram-dependent universality-breaking matrix elements which will not appear in integrated and weighted hadronic cross sections. However, it could be that they contribute to the second or higher transverse moments.

5. Parametrizations of parton correlators

At leading twist the parametrizations of the different TMD quark correlators are given by

$$\begin{aligned} \Phi^{(T\text{-even})}(x, p_T) = & \frac{1}{2} \left\{ f_1(x, p_T^2) \not{p} + \frac{1}{2} h_{1T}(x, p_T^2) \gamma_5 [\not{S}_T, \not{p}] \right. \\ & + S_L h_{1L}^\perp(x, p_T^2) \gamma_5 \frac{[\not{p}_T, \not{p}]}{2M} + \frac{\mathbf{p}_T \cdot \mathbf{S}_T}{M} h_{1T}^\perp(x, p_T^2) \gamma_5 \frac{[\not{p}_T, \not{p}]}{2M} \\ & \left. + S_L g_{1L}(x, p_T^2) \gamma_5 \not{p} + \frac{\mathbf{p}_T \cdot \mathbf{S}_T}{M} g_{1T}(x, p_T^2) \gamma_5 \not{p} \right\}, \quad (25a) \end{aligned}$$

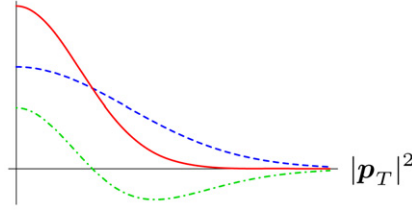


Fig. 2. Possible behavior of the universal distribution function $f_1(x, p_T^2)$ (dashed line) and the process-dependent function $f_1^{(ab \rightarrow cd)}(x, p_T^2)$ (solid line) as a function of $|p_T|^2$. Their difference-function, $\delta f_1^{(ab \rightarrow cd)}(x, p_T^2)$ (dash-dotted line), vanishes upon integration over p_T .

$$\Phi^{(T\text{-odd})}(x, p_T) = \frac{1}{2} \left\{ i h_1^\perp(x, p_T^2) \frac{[\not{p}_T, \not{\epsilon}]}{2M} - \frac{\epsilon_T^{p_T S_T}}{M} f_{1T}^\perp(x, p_T^2) \not{\epsilon} \right\}, \quad (25b)$$

and

$$\begin{aligned} \delta \Phi^{(ab \rightarrow cd)}(x, p_T) &= \frac{1}{2} \left\{ \delta f_1^{(ab \rightarrow cd)}(x, p_T^2) \not{\epsilon} + \frac{1}{2} \delta h_{1T}^{(ab \rightarrow cd)}(x, p_T^2) \gamma_5 [\not{\epsilon}_T, \not{\epsilon}] \right. \\ &\quad + S_L \delta h_{1L}^{\perp(ab \rightarrow cd)}(x, p_T^2) \gamma_5 \frac{[\not{p}_T, \not{\epsilon}]}{2M} + \frac{\mathbf{p}_T \cdot \mathbf{S}_T}{M} \delta h_{1T}^{\perp(ab \rightarrow cd)}(x, p_T^2) \gamma_5 \frac{[\not{p}_T, \not{\epsilon}]}{2M} \\ &\quad + S_L \delta g_{1L}^{(ab \rightarrow cd)}(x, p_T^2) \gamma_5 \not{\epsilon} + \frac{\mathbf{p}_T \cdot \mathbf{S}_T}{M} \delta g_{1T}^{(ab \rightarrow cd)}(x, p_T^2) \gamma_5 \not{\epsilon} \\ &\quad \left. + i \delta h_1^{\perp(ab \rightarrow cd)}(x, p_T^2) \frac{[\not{p}_T, \not{\epsilon}]}{2M} - \frac{\epsilon_T^{p_T S_T}}{M} \delta f_{1T}^{\perp(ab \rightarrow cd)}(x, p_T^2) \not{\epsilon} \right\}, \quad (25c) \end{aligned}$$

with similar parametrizations for the matrix elements $\pi \delta \Phi_G^{(ab \rightarrow cd)}(x, p_T)$ containing a (different) set of distribution functions $\delta f_1^{([a]b \rightarrow cd)}$, etc. The quark distribution functions that appear in the parametrizations (25a) and (25b) are the familiar T -even and T -odd (respectively) quark distribution functions as measured in SIDIS. On the other hand, the quark distribution functions in (25c) and in the parametrization of $\delta \Phi_G$ are process dependent. From the properties in (20) one finds that the functions δf_1 , δh_{1T} and δg_{1L} vanish upon p_T -integration, for instance

$$\int d^2 p_T \delta f_1^{(ab \rightarrow cd)}(x, p_T^2) = 0, \quad (26)$$

illustrated in Fig. 2. Also the functions $\delta h_{1L}^{\perp(1)}$, $\delta h_{1T}^{\perp(1)}$, $\delta g_{1T}^{(1)}$, $\delta h_1^{\perp(1)}$ and $\delta f_{1T}^{\perp(1)}$ vanish, e.g.,

$$\int d^2 p_T \frac{\mathbf{p}_T^2}{2M^2} \delta f_{1T}^{\perp(ab \rightarrow cd)}(x, p_T^2) = 0. \quad (27)$$

The same holds for the corresponding functions in the parametrization of $\pi \delta \Phi_G^{(ab \rightarrow cd)}(x, p_T)$.

For gluon distribution correlators we use the parameterizations of Ref. [36] with the naming convention discussed in Ref. [37]:

$$\Gamma^{(T\text{-even})\mu\nu}(x, p_T) = \frac{1}{2x} \left\{ -g_T^{\mu\nu} f_1^g(x, p_T^2) + \left(\frac{p_T^\mu p_T^\nu}{M^2} + g_T^{\mu\nu} \frac{p_T^2}{2M^2} \right) h_1^{\perp g}(x, p_T^2) \right. \\ \left. + i\epsilon_T^{\mu\nu} S_L g_{1L}^g(x, p_T^2) + i\epsilon_T^{\mu\nu} \frac{p_T \cdot S_T}{M} g_{1T}^g(x, p_T^2) \right\}, \quad (28a)$$

$$\Gamma_{(f/d)}^{(T\text{-odd})\mu\nu}(x, p_T) \\ = \frac{1}{2x} \left\{ g_T^{\mu\nu} \frac{\epsilon_T^{p_T S_T}}{M} f_{1T}^{\perp g(f/d)}(x, p_T^2) - \frac{\epsilon_T^{p_T\{\mu} S_T^{\nu\}} + \epsilon_T^{S_T\{\mu} p_T^{\nu\}}}{4M} h_{1T}^{g(f/d)}(x, p_T^2) \right. \\ \left. - \frac{\epsilon_T^{p_T\{\mu} p_T^{\nu\}}}{2M^2} S_L h_{1L}^{\perp g(f/d)}(x, p_T^2) - \frac{\epsilon_T^{p_T\{\mu} p_T^{\nu\}}}{2M^2} \frac{p_T \cdot S_T}{M} h_{1T}^{\perp g(f/d)}(x, p_T^2) \right\}. \quad (28b)$$

In particular one has two distinct gluon-Sivers distribution functions $f_{1T}^{\perp g(f)}(x, p_T^2)$ and $f_{1T}^{\perp g(d)}(x, p_T^2)$ corresponding to the two ways to construct T -odd gluon correlators. Their first transverse moments are the functions $G_T^{(f/d)(1)}(x) \equiv f_{1T}^{\perp g(f/d)(1)}(x)$ introduced in Ref. [19]. The matrix elements $\delta\Gamma^{(ab \rightarrow cd)}(x, p_T^2)$ and $\pi\delta\Gamma_{G_{f/d}}^{(ab \rightarrow cd)}(x, p_T^2)$ contain both T -even and T -odd gluon distribution functions. These functions, however, will vanish under p_T -integration or weighting, as for the quark universality-breaking distribution functions (cf. (26) and (27)).

The analysis above can be extended to antiquark distribution correlators in the obvious way. Also the treatment of (anti)quark fragmentation correlators is straightforward, with the understanding that in the matrix elements $\Delta^{(T\text{-even/odd})}(z, k_T)$ the superscripts T -even and T -odd refer to the operator structure in the correlators, each of which contain both T -even and T -odd fragmentation functions (this also holds for the gluon fragmentation correlators \hat{F}).

6. Transverse momentum dependent functions in hadronic cross sections

The advantage of the decomposition of the TMD quark (and gluon) correlators in expression (15) (and (23)) is evident: instead of having to list the TMD correlators $\Phi^{[D]}$ ($\Gamma^{[D]}$) for every cut Feynman diagram of a process, the two (or three in the case of gluons) matrix elements on the r.h.s. of those decompositions suffice for that particular process. This observation leads to a reduction in the number of TMD matrix elements that need to be considered. From a book-keeping point of view this makes the treatment of TMD correlators more manageable. What is perhaps more important is that the decompositions (15) and (23) will allow us to show that also unweighted, unintegrated hadronic cross sections can be written as products of soft parton correlators and hard partonic functions that are separately manifestly gauge invariant, analogous to the collinear case in expressions (12) and (13). Moreover, the hard partonic functions are the partonic cross sections or the gluonic pole cross sections. This is what will be argued in this section. The proper context should be within a transverse momentum dependent factorization theorem. For most processes such a theorem does not exist yet. We will therefore take as a starting point the assumption that the hadronic cross sections will factorize in a hard partonic function and a soft parton correlator for each observed hadron separately, and that gluon initial and final-state interactions will lead to the required Wilson lines. We believe that this assumption is sufficiently generic for our conclusions to be applicable for many hadronic processes, in particular to back-to-back dijet or photon-jet production in proton–proton scattering.

By inserting the decompositions (15) and (23) of the TMD parton correlators into the expression for the unintegrated hadronic cross section in Eq. (11) the parton contribution to a hard

$2 \rightarrow 2$ process becomes

$$\begin{aligned}
 \Sigma(p_1, p_2, k_1, k_2) &= \sum_{a, \dots, d} \left\{ \text{Tr} \left[\Phi_a^{(ab \rightarrow cd)}(x_1, p_{1T}) \Phi_b^{(ab \rightarrow cd)}(x_2, p_{2T}) \hat{\Sigma}_{ab \rightarrow cd} \Delta_c^{(ab \rightarrow cd)}(z_1, k_{1T}) \right. \right. \\
 &\quad \times \Delta_d^{(ab \rightarrow cd)}(z_2, k_{2T}) \left. \right] \\
 &\quad + \text{Tr} \left[\pi \Phi_{aG}^{(ab \rightarrow cd)}(x_1, p_{1T}) \Phi_b^{(ab \rightarrow cd)}(x_2, p_{2T}) \hat{\Sigma}_{[a]b \rightarrow cd} \Delta_c^{(ab \rightarrow cd)}(z_1, k_{1T}) \right. \\
 &\quad \times \Delta_d^{(ab \rightarrow cd)}(z_2, k_{2T}) \left. \right] + \dots \left. \right\}, \tag{29}
 \end{aligned}$$

which forms the central result of this paper. Again it is implicitly implied that parton a is an (anti)quark, since if it were a gluon there would be two gluonic pole terms $\Gamma_{G_f} \hat{\Sigma}_{[g]b \rightarrow cd}^{(f)}$ and $\Gamma_{G_d} \hat{\Sigma}_{[g]b \rightarrow cd}^{(d)}$. In (29) both terms in the decomposition of the TMD correlator of (anti)quark a have been given explicitly, while only the first terms of the decompositions (15) and (23) were used for the other partons. The “ $+\dots$ ” contains the other possible combinations of the terms in those decompositions and also the contributions where parton a is a gluon.

The hard functions in expression (29) are the partonic and gluonic pole cross sections in (14a) and (14b) (in the collinear expansions of the hard functions, which have corrections at order $\mathcal{O}(1/s)$). Hence, the expression in (29) demonstrates that by using the decompositions in (15) and (23) and by introducing the gluonic pole cross sections, also the unintegrated, unweighted hadronic cross section can be written as a product of soft TMD parton correlators and hard partonic functions that are separately and manifestly gauge invariant. After performing the traces these hard functions reduce to the partonic and gluonic pole cross sections calculated in Refs. [17, 19, 28]. Hence, it follows that the gluonic pole cross sections that have been seen [17, 19, 27, 28] to represent the hard partonic functions in weighted spin asymmetries already appear in the fully TMD cross sections. This conclusion is consistent with the work in Refs. [20, 21], where a TMD factorization formula for the quark-Sivers contribution to single transverse-spin asymmetries in dijet production is proposed with the gluonic pole cross sections of Refs. [17, 19] as hard functions. However, the work in Refs. [20, 21] limits to one-gluon exchange and as a result it obtains the distribution functions measured in SIDIS (i.e., transverse momentum dependent distribution functions with a future pointing Wilson line in the hadronic matrix elements defining them). In contrast, our expression (29) involves correlators with process dependent Wilson lines in the operator definitions and a factorized form with universal distribution functions would only be achieved if, regardless of the appearance of process dependent Wilson lines in their definitions, the TMD matrix elements in (29) are identical for all partonic channels and equal those in semi-inclusive deep inelastic scattering. In the present context this translates into a vanishing of all universality-breaking matrix elements, for which we see no reason. Indeed, universality has recently also been disputed in Ref. [22], where it is argued that a TMD factorization theorem for this process with universal distribution functions is not possible. This is confirmed by an explicit calculation including the exchange of two collinear gluons [25]. A recent extension [24] of the work in Refs. [20, 21] including two-gluon exchange also points to non-universality. Moreover, it shows that Refs. [16–19], [20, 21] and [22] are consistent to (at least) that order. We want to emphasize that for a full connection the role of soft factors, which have been neglected in the present study, should also be investigated.

With the parametrizations (25) and (28) inserted in (29), an expression for the hadronic cross section in terms of TMD parton distribution and fragmentation functions is obtained. After performing the traces these hard functions reduce to the partonic and gluonic pole cross sections

encountered in Refs. [17,19,27,28]. As an illustration we consider the contribution to (29) of an unpolarized quark in an unpolarized hadron, important for unintegrated spin averaged hadronic cross sections (summations over parton types are understood):

$$d\sigma_U \sim \underbrace{\left\{ f_1(x_1, p_{1T}^2) + \delta f_1^{(ab \rightarrow cd)}(x_1, p_{1T}^2) \right\}}_{f_1^{(ab \rightarrow cd)}(x_1, p_{1T}^2)} d\hat{\sigma}_{ab \rightarrow cd} + \delta f_1^{([a]b \rightarrow cd)}(x_1, p_{1T}^2) d\hat{\sigma}_{[a]b \rightarrow cd}. \quad (30a)$$

Taking as a second example the contribution of an unpolarized quark in a transversally polarized hadron, important for unintegrated single-spin asymmetries, one finds

$$d\sigma_T \sim \delta f_{1T}^{\perp(ab \rightarrow cd)}(x_1, p_{1T}^2) d\hat{\sigma}_{ab \rightarrow cd} + \underbrace{\left\{ f_{1T}^{\perp}(x_1, p_{1T}^2) + \delta f_{1T}^{\perp([a]b \rightarrow cd)}(x_1, p_{1T}^2) \right\}}_{f_{1T}^{\perp(ab \rightarrow cd)}(x_1, p_{1T}^2)} d\hat{\sigma}_{[a]b \rightarrow cd}. \quad (30b)$$

The terms with only universal T -even functions will appear folded with partonic cross sections and the universal T -odd functions with gluonic pole cross sections. In addition, there are various universality-breaking functions that appear with partonic cross sections or gluonic pole cross sections. A shorter notation for some of the terms in these expressions could have been obtained by not extracting the universal T -even and T -odd parts of the distribution functions f_1 and f_{1T}^{\perp} , as indicated by the underbraces in (30). In particular, due to these universality-breaking functions the gluonic pole cross sections also appear in the unweighted spin-averaged cross sections (30a) and the usual partonic cross sections also appear in the unweighted single-spin asymmetries (30b). However, in the light of the properties in (26)–(27) it is seen that terms with universality-breaking matrix elements do not contribute to the integrated and q_T -weighted (where q_T is as defined in Section 3) hadronic cross sections which are expressed in terms of the structures in (12) and (13):

$$\text{integrated: } \langle d\sigma \rangle \sim \Sigma(x_1, x_2) \propto f_1(x_1) d\hat{\sigma}_{ab \rightarrow cd}, \quad (31a)$$

$$\text{weighted: } \langle q_T d\sigma \rangle \sim (\Sigma_{1\partial} + \Sigma_{2\partial})(x_1, x_2) \propto f_{1T}^{\perp(1)}(x_1) d\hat{\sigma}_{[a]b \rightarrow cd}. \quad (31b)$$

For back-to-back jet production in polarized proton–proton scattering ($p^\uparrow p \rightarrow jjX$) this (in essence) reproduces the results of Refs. [17,19,27], while for photon-jet production ($p^\uparrow p \rightarrow \gamma jX$) it reproduces the results in Ref. [28].

Only if the universality-breaking matrix elements vanish in the unintegrated, unweighted cross sections (29) and (30) does one also in the TMD case arrive at the situation with universal functions only. Otherwise, the non-universality of the unweighted processes is important. It will affect the results of experiments that try to look at explicit p_T -dependence or that construct weighted cross sections involving convolutions of TMD functions which upon integration do not factorize into transverse moments, e.g. when looking at $\langle \sin(\phi_h \pm \phi_S) \rangle$, rather than $\langle P_{\pi\perp} \sin(\phi_h \pm \phi_S) \rangle$ asymmetries in SIDIS to extract transversity or Sivers functions.

7. Summary

We have argued that the gluonic pole cross sections, the hard partonic scattering functions that are folded with the collinear parton distribution functions in weighted spin-asymmetries, also appear in unintegrated, unweighted hadronic cross sections. Assuming as a starting point that the hadronic cross section factorizes at the diagrammatic level in a hard partonic function

and for each of the observed hadrons a soft parton correlator, we have shown that the transverse momentum dependent cross section can be written in terms of soft and hard functions that are separately manifestly gauge-invariant. The hard functions are the partonic cross sections or the gluonic pole cross sections. The latter do not show up in the integrated cross sections. The soft functions are the TMD parton distribution (and fragmentation) functions with Wilson lines on the light-front in their field theoretical operator definitions. These Wilson lines can be considered as the collective effect of gluon initial and final state interactions. Since they are process dependent, the TMD parton distributions are in general non-universal. By systematically separating the universality-breaking parts from the universal T -even and T -odd parts, we arrived at an expression that has soft parts multiplying the partonic and gluonic pole cross sections, as was also found in Refs. [20,21]. However, in the gauge link approach taken here the gluonic pole cross section can also emerge in TMD spin-averaged processes and ordinary partonic cross sections can also arise in TMD single-spin asymmetries. They appear with universality-breaking functions that will vanish for the integrated and weighted processes considered in Refs. [17,19,27,28]. They also vanish at the level of one-gluon radiation contributions, which corresponds to the order g term of the Wilson lines.

The non-universal terms are well-defined matrix elements. All universality-breaking matrix elements that are encountered at tree-level in proton–proton scattering with $2 \rightarrow 2$ partonic processes have been calculated and are given in Appendix A. In particular, the process-dependent universality-breaking matrix elements $\delta\Phi$, etc., disappear in the simple electroweak processes with underlying hard parts like $q\gamma^* \rightarrow q$ and $q\bar{q} \rightarrow \gamma^*$. We believe that the explicit identification of universality-breaking matrix elements is an important contribution to arrive at a unified picture of TMD factorization of hadronic scattering processes.

Acknowledgements

We are grateful to Alessandro Bacchetta, Daniël Boer, John Collins, Umberto D’Alesio, Leonard Gamberg, Andreas Metz, Peter Schweitzer, Werner Vogelsang and Feng Yuan for useful discussions. The work of C.B. was supported by the foundation for Fundamental Research of Matter (FOM) and the National Organization for Scientific Research (NWO).

Appendix A. Universality-breaking matrix elements

We list the universality-breaking matrix elements that appear at tree-level in proton–proton scattering. To improve readability we employ the schematic notation $(2\pi)^{-3} \int d(\xi \cdot P) d^2\xi_T e^{ip \cdot \xi} \langle P, S | \bar{\psi}(0) \mathcal{U}\psi(\xi) | P, S \rangle_{\text{LF}} \propto \langle \bar{\psi}(0) \mathcal{U}\psi(\xi) \rangle$, and similarly for the other parton correlators. For the $2 \rightarrow 2$ partonic channels with four colored external legs we only list the quark and/or gluon distribution correlators. As can be seen from the tables in Ref. [18] the TMD correlators of the other partons are very similar in structure to the correlators considered here and can straightforwardly be obtained by comparing the results below to the tables of the stated reference. In particular the correlators in $\bar{q}g \rightarrow \bar{q}g$ scattering are simply obtained by comparing to those in $qg \rightarrow qg$ scattering. Similarly, the fragmentation correlators in $q\bar{q} \rightarrow gg$ ($gg \rightarrow q\bar{q}$) scattering can be obtained by comparing to the distribution correlators in $gg \rightarrow q\bar{q}$ ($q\bar{q} \rightarrow gg$).

$$\boxed{q\gamma^* \rightarrow q, \quad q\bar{q} \rightarrow \gamma^*}$$

$$\delta\Phi^{(q\gamma^* \rightarrow q)}(x, p_T) = \pi \delta\Phi_G^{(q\gamma^* \rightarrow q)}(x, p_T) = 0, \quad (\text{A.1})$$

$$\delta\Phi^{(q\bar{q}\rightarrow\gamma^*)}(x, p_T) = \pi\delta\Phi_G^{(q\bar{q}\rightarrow\gamma^*)}(x, p_T) = 0. \quad (\text{A.2})$$

$$\boxed{q\bar{q} \rightarrow g\gamma}$$

$$\delta\Phi^{(q\bar{q}\rightarrow g\gamma)}(x, p_T) = \pi\delta\Phi_G^{(q\bar{q}\rightarrow g\gamma)}(x, p_T) \propto \left\langle \bar{\psi}(0) \left\{ \frac{1}{2} \frac{\text{Tr}[\mathcal{U}^{[\square]\dagger}]}{N} \mathcal{U}^{[+]} - \frac{1}{2} \mathcal{U}^{[+]} \right\} \psi(\xi) \right\rangle, \quad (\text{A.3})$$

$$\delta\bar{\Phi}^{(q\bar{q}\rightarrow g\gamma)}(x, p_T) = \pi\delta\bar{\Phi}_G^{(q\bar{q}\rightarrow g\gamma)}(x, p_T) \propto \left\langle \bar{\psi}(0) \left\{ \frac{1}{2} \frac{\text{Tr}[\mathcal{U}^{[\square]}]}{N} \mathcal{U}^{[+]\dagger} - \frac{1}{2} \mathcal{U}^{[+]\dagger} \right\} \psi(\xi) \right\rangle, \quad (\text{A.4})$$

$$\delta\hat{\Gamma}^{(qg\rightarrow q\gamma)}(z, k_T) = \pi\delta\hat{\Gamma}_{G_f}^{(qg\rightarrow q\gamma)}(z, k_T) = \pi\delta\hat{\Gamma}_{G_d}^{(qg\rightarrow q\gamma)}(z, k_T) = 0. \quad (\text{A.5})$$

$$\boxed{qg \rightarrow q\gamma}$$

$$\delta\Phi^{(qg\rightarrow q\gamma)}(x, p_T) = -\pi\delta\Phi_G^{(qg\rightarrow q\gamma)}(x, p_T) \propto \left\langle \bar{\psi}(0) \left\{ \frac{1}{2} \frac{\text{Tr}[\mathcal{U}^{[\square]}]}{N} \mathcal{U}^{[-]} - \frac{1}{2} \mathcal{U}^{[-]} \right\} \psi(\xi) \right\rangle, \quad (\text{A.6})$$

$$\delta\Delta^{(qg\rightarrow q\gamma)}(z, k_T) = \pi\delta\Delta_G^{(qg\rightarrow q\gamma)}(z, k_T) = 0, \quad (\text{A.7})$$

$$\begin{aligned} \delta\Gamma^{(qg\rightarrow q\gamma)}(x, p_T) \\ \propto \left\langle \text{Tr} \left[F(0) \mathcal{U}^{[+]} F(\xi) \left\{ \frac{1}{2} \mathcal{U}^{[-]\dagger} - \frac{1}{2} \mathcal{U}^{[+]\dagger} \right\} + F(0) \mathcal{U}^{[-]} F(\xi) \left\{ \frac{1}{2} \mathcal{U}^{[+]\dagger} - \frac{1}{2} \mathcal{U}^{[-]\dagger} \right\} \right] \right\rangle, \end{aligned} \quad (\text{A.8})$$

$$\pi\delta\Gamma_{G_f}^{(qg\rightarrow q\gamma)}(x, p_T) = \pi\delta\Gamma_{G_d}^{(qg\rightarrow q\gamma)}(x, p_T) = 0. \quad (\text{A.9})$$

$$\boxed{qq \rightarrow qq}$$

$$\delta\Phi^{(qq\rightarrow qq)}(x, p_T) \propto \left\langle \bar{\psi}(0) \left\{ \frac{3}{2} \frac{\text{Tr}[\mathcal{U}^{[\square]}]}{N} \mathcal{U}^{[+]} - \frac{1}{2} \mathcal{U}^{[\square]} \mathcal{U}^{[+]} - \frac{1}{2} \mathcal{U}^{[+]} - \frac{1}{2} \mathcal{U}^{[-]} \right\} \psi(\xi) \right\rangle, \quad (\text{A.10})$$

$$\pi\delta\Phi_G^{(qq\rightarrow qq)}(x, p_T) \propto \left\langle \bar{\psi}(0) \left\{ \frac{1}{2} \mathcal{U}^{[\square]} \mathcal{U}^{[+]} - \frac{1}{2} \frac{\text{Tr}[\mathcal{U}^{[\square]}]}{N} \mathcal{U}^{[+]} - \frac{1}{2} \mathcal{U}^{[+]} + \frac{1}{2} \mathcal{U}^{[-]} \right\} \psi(\xi) \right\rangle. \quad (\text{A.11})$$

$$\boxed{q\bar{q} \rightarrow q\bar{q}}$$

$$\delta\Phi^{(q\bar{q}\rightarrow q\bar{q})}(x, p_T) = \pi\delta\Phi_G^{(q\bar{q}\rightarrow q\bar{q})}(x, p_T) \propto \left\langle \bar{\psi}(0) \left\{ \frac{1}{2} \frac{\text{Tr}[\mathcal{U}^{[\square]\dagger}]}{N} \mathcal{U}^{[+]} - \frac{1}{2} \mathcal{U}^{[+]} \right\} \psi(\xi) \right\rangle \quad (\text{A.12})$$

$$\boxed{qg \rightarrow qg}$$

$$\begin{aligned} \delta\Phi^{(qg\rightarrow qg)}(x, p_T) \propto \left\langle \bar{\psi}(0) \left\{ \frac{3}{2} \frac{N^2 + 1}{N^2 - 1} \frac{\text{Tr}[\mathcal{U}^{[\square]}]}{N} \frac{\text{Tr}[\mathcal{U}^{[\square]\dagger}]}{N} \mathcal{U}^{[+]} - \frac{1}{2} \frac{N^2 + 1}{N^2 - 1} \mathcal{U}^{[+]} \right. \right. \\ \left. \left. + \frac{1}{2} \frac{\text{Tr}[\mathcal{U}^{[\square]}]}{N} \mathcal{U}^{[-]} - \frac{1}{2} \mathcal{U}^{[-]} \right\} \psi(\xi) \right\rangle, \end{aligned} \quad (\text{A.13})$$

$$\pi \delta \Phi_G^{(qg \rightarrow qg)}(x, p_T) \propto \left\langle \bar{\psi}(0) \left\{ \frac{1}{2} \frac{\text{Tr}[\mathcal{U}^{[\square]}]}{N} \frac{\text{Tr}[\mathcal{U}^{[\square]\dagger}]}{N} \mathcal{U}^{[+]} - \frac{1}{2} \mathcal{U}^{[+]} \right. \right. \\ \left. \left. - \frac{1}{2} \frac{\text{Tr}[\mathcal{U}^{[\square]}]}{N} \mathcal{U}^{[-]} + \frac{1}{2} \mathcal{U}^{[-]} \right\} \psi(\xi) \right\rangle, \quad (\text{A.14})$$

$$\delta \Gamma^{(qg \rightarrow qg)}(x, p_T) \propto \left\langle \text{Tr} \left[F(0) \mathcal{U}^{[+]} F(\xi) \left\{ \frac{1}{2} \frac{\text{Tr}[\mathcal{U}^{[\square]\dagger}]}{N} \mathcal{U}^{[-\dagger]} - \frac{1}{2} \mathcal{U}^{[-\dagger]} \right\} \right. \right. \\ \left. \left. + F(0) \mathcal{U}^{[-]} F(\xi) \left\{ \frac{1}{2} \frac{\text{Tr}[\mathcal{U}^{[\square]}]}{N} \mathcal{U}^{[+\dagger]} - \frac{1}{2} \mathcal{U}^{[+\dagger]} \right\} \right] \right\rangle, \quad (\text{A.15})$$

$$\pi \delta \Gamma_{G_f}^{(qg \rightarrow qg)}(x, p_T) \propto \left\langle \text{Tr} \left[F(0) \mathcal{U}^{[+]} F(\xi) \left\{ \frac{1}{2} \frac{\text{Tr}[\mathcal{U}^{[\square]\dagger}]}{N} \mathcal{U}^{[-\dagger]} - \frac{1}{2} \mathcal{U}^{[-\dagger]} \right\} \right. \right. \\ \left. \left. - F(0) \mathcal{U}^{[-]} F(\xi) \left\{ \frac{1}{2} \frac{\text{Tr}[\mathcal{U}^{[\square]}]}{N} \mathcal{U}^{[+\dagger]} - \frac{1}{2} \mathcal{U}^{[+\dagger]} \right\} \right] \right\rangle, \quad (\text{A.16})$$

$$\pi \delta \Gamma_{G_d}^{(qg \rightarrow qg)}(x, p_T) \propto \left\langle \text{Tr} \left[F(0) \mathcal{U}^{[+]} F(\xi) \left\{ \frac{1}{2} \mathcal{U}^{[-\dagger]} - \frac{1}{2} \frac{\text{Tr}[\mathcal{U}^{[\square]}]}{N} \mathcal{U}^{[+\dagger]} \right\} \right. \right. \\ \left. \left. + F(0) \mathcal{U}^{[-]} F(\xi) \left\{ \frac{1}{2} \mathcal{U}^{[+\dagger]} - \frac{1}{2} \frac{\text{Tr}[\mathcal{U}^{[\square]\dagger}]}{N} \mathcal{U}^{[-\dagger]} \right\} \right] \right\rangle. \quad (\text{A.17})$$

$$\boxed{q\bar{q} \rightarrow gg}$$

$$\delta \Phi^{(q\bar{q} \rightarrow gg)}(x, p_T) = \pi \delta \Phi_G^{(q\bar{q} \rightarrow gg)}(x, p_T) \propto \left\langle \bar{\psi}(0) \left\{ \frac{1}{2} \frac{\text{Tr}[\mathcal{U}^{[\square]\dagger}]}{N} \mathcal{U}^{[+]} - \frac{1}{2} \mathcal{U}^{[+]} \right\} \psi(\xi) \right\rangle. \quad (\text{A.18})$$

$$\boxed{gg \rightarrow q\bar{q}}$$

$$\delta \Gamma^{(gg \rightarrow q\bar{q})}(x, p_T) \\ = -\pi \delta \Gamma_{G_f}^{(gg \rightarrow q\bar{q})}(x, p_T) \\ \propto \left\langle \text{Tr} \left[F(0) \mathcal{U}^{[+]} F(\xi) \left\{ \frac{1}{2} \frac{\text{Tr}[\mathcal{U}^{[\square]\dagger}]}{N} \mathcal{U}^{[-\dagger]} - \frac{1}{2} \mathcal{U}^{[+\dagger]} \right\} \right. \right. \\ \left. \left. + F(0) \mathcal{U}^{[-]} F(\xi) \left\{ \frac{1}{2} \frac{\text{Tr}[\mathcal{U}^{[\square]}]}{N} \mathcal{U}^{[+\dagger]} - \frac{1}{2} \mathcal{U}^{[-\dagger]} \right\} \right] \right\rangle, \quad (\text{A.19})$$

$$\pi \delta \Gamma_{G_d}^{(gg \rightarrow q\bar{q})}(x, p_T) \propto \left\langle \text{Tr} \left[F(0) \mathcal{U}^{[+]} F(\xi) \left\{ \frac{1}{2} \frac{\text{Tr}[\mathcal{U}^{[\square]\dagger}]}{N} \mathcal{U}^{[-\dagger]} - \frac{1}{2} \mathcal{U}^{[-\dagger]} \right\} \right. \right. \\ \left. \left. - F(0) \mathcal{U}^{[-]} F(\xi) \left\{ \frac{1}{2} \frac{\text{Tr}[\mathcal{U}^{[\square]}]}{N} \mathcal{U}^{[+\dagger]} - \frac{1}{2} \mathcal{U}^{[+\dagger]} \right\} \right] \right\rangle. \quad (\text{A.20})$$

$$\boxed{gg \rightarrow gg}$$

$$\delta \Gamma^{(gg \rightarrow gg)}(x, p_T) \propto \left\langle \text{Tr} \left[F(0) \mathcal{U}^{[+]} F(\xi) \left\{ \frac{1}{2} \frac{\text{Tr}[\mathcal{U}^{[\square]\dagger}]}{N} \mathcal{U}^{[-\dagger]} - \frac{1}{2} \mathcal{U}^{[+\dagger]} \right\} \right. \right. \\ \left. \left. + F(0) \mathcal{U}^{[-]} F(\xi) \left\{ \frac{1}{2} \frac{\text{Tr}[\mathcal{U}^{[\square]}]}{N} \mathcal{U}^{[+\dagger]} - \frac{1}{2} \mathcal{U}^{[-\dagger]} \right\} \right] \right\rangle, \quad (\text{A.21})$$

$$\begin{aligned}
& \pi \delta \Gamma_{G_f}^{(gg \rightarrow gg)}(x, p_T) \\
& \propto \left\langle \text{Tr} \left[\frac{1}{N^2} F(\xi) \mathcal{U}^{[\square]^\dagger} \mathcal{U}^{[+]^\dagger} F(0) \mathcal{U}^{[\square]} \mathcal{U}^{[+]} + F(0) \mathcal{U}^{[+]} F(\xi) \mathcal{U}^{[+]^\dagger} \frac{\text{Tr}[\mathcal{U}^{[\square]}]}{N} \frac{\text{Tr}[\mathcal{U}^{[\square]^\dagger}]}{N} \right. \right. \\
& \quad + F(0) \mathcal{U}^{[+]} F(\xi) \left\{ \frac{N^2 + 4}{2N^2} \mathcal{U}^{[+]^\dagger} - \frac{1}{2} \frac{\text{Tr}[\mathcal{U}^{[\square]^\dagger}]}{N} \mathcal{U}^{[-]^\dagger} \right\} \\
& \quad \left. \left. + F(0) \mathcal{U}^{[-]} F(\xi) \left\{ \frac{N^2 + 2}{2N^2} \mathcal{U}^{[-]^\dagger} - \frac{1}{2} \frac{\text{Tr}[\mathcal{U}^{[\square]}]}{N} \mathcal{U}^{[+]^\dagger} \right\} \right] \right\rangle, \tag{A.22}
\end{aligned}$$

$$\pi \delta \Gamma_{G_d}^{(gg \rightarrow gg)}(x, p_T) = 0. \tag{A.23}$$

References

- [1] A.V. Efremov, O.V. Teryaev, Sov. J. Nucl. Phys. 36 (1982) 140.
- [2] A.V. Efremov, O.V. Teryaev, Phys. Lett. B 150 (1985) 383.
- [3] J.-W. Qiu, G. Sterman, Phys. Rev. Lett. 67 (1991) 2264.
- [4] J.-W. Qiu, G. Sterman, Nucl. Phys. B 378 (1992) 52.
- [5] J.-W. Qiu, G. Sterman, Phys. Rev. D 59 (1999) 014004, hep-ph/9806356.
- [6] Y. Kanazawa, Y. Koike, Phys. Lett. B 478 (2000) 121, hep-ph/0001021.
- [7] H. Eguchi, Y. Koike, K. Tanaka, Nucl. Phys. B 763 (2007) 198, hep-ph/0610314.
- [8] Y. Koike, K. Tanaka, Phys. Lett. B 646 (2007) 232, hep-ph/0612117.
- [9] D.W. Sivers, Phys. Rev. D 41 (1990) 83.
- [10] D.W. Sivers, Phys. Rev. D 43 (1991) 261.
- [11] D. Boer, P.J. Mulders, Phys. Rev. D 57 (1998) 5780, hep-ph/9711485.
- [12] S.J. Brodsky, D.S. Hwang, I. Schmidt, Phys. Lett. B 530 (2002) 99, hep-ph/0201296.
- [13] S.J. Brodsky, D.S. Hwang, I. Schmidt, Nucl. Phys. B 642 (2002) 344, hep-ph/0206259.
- [14] J.C. Collins, Phys. Lett. B 536 (2002) 43, hep-ph/0204004.
- [15] D. Boer, P.J. Mulders, F. Pijlman, Nucl. Phys. B 667 (2003) 201, hep-ph/0303034.
- [16] C.J. Bomhof, P.J. Mulders, F. Pijlman, Phys. Lett. B 596 (2004) 277, hep-ph/0406099.
- [17] A. Bacchetta, C.J. Bomhof, P.J. Mulders, F. Pijlman, Phys. Rev. D 72 (2005) 034030, hep-ph/0505268.
- [18] C.J. Bomhof, P.J. Mulders, F. Pijlman, Eur. Phys. J. C 47 (2006) 147, hep-ph/0601171.
- [19] C.J. Bomhof, P.J. Mulders, JHEP 0702 (2007) 029, hep-ph/0609206.
- [20] J.-W. Qiu, W. Vogelsang, F. Yuan, Phys. Lett. B 650 (2007) 373, arXiv: 0704.1153 [hep-ph].
- [21] J.-W. Qiu, W. Vogelsang, F. Yuan, arXiv: 0706.1196 [hep-ph].
- [22] J. Collins, J.-W. Qiu, Phys. Rev. D 75 (2007) 114014, arXiv: 0705.2141 [hep-ph].
- [23] P.G. Ratcliffe, O.V. Teryaev, hep-ph/0703293.
- [24] W. Vogelsang, F. Yuan, arXiv: 0708.4398 [hep-ph].
- [25] J. Collins, arXiv: 0708.4410 [hep-ph].
- [26] D. Boer, P.J. Mulders, Nucl. Phys. B 569 (2000) 505, hep-ph/9906223.
- [27] C.J. Bomhof, P.J. Mulders, W. Vogelsang, F. Yuan, Phys. Rev. D 75 (2007) 074019, hep-ph/0701277.
- [28] A. Bacchetta, C. Bomhof, U. D'Alesio, P.J. Mulders, F. Murgia, hep-ph/0703153.
- [29] A.V. Efremov, A.V. Radyushkin, Theor. Math. Phys. 44 (1981) 774.
- [30] A.V. Belitsky, X. Ji, F. Yuan, Nucl. Phys. B 656 (2003) 165, hep-ph/0208038.
- [31] X.-D. Ji, Phys. Lett. B 289 (1992) 137.
- [32] X.-D. Ji, J.-P. Ma, F. Yuan, Phys. Rev. D 71 (2005) 034005, hep-ph/0404183.
- [33] X.-D. Ji, J.-P. Ma, F. Yuan, Phys. Lett. B 597 (2004) 299, hep-ph/0405085.
- [34] D. Boer, W. Vogelsang, Phys. Rev. D 69 (2004) 094025, hep-ph/0312320.
- [35] C.J. Bomhof, Azimuthal spin asymmetries in hadronic processes, Ph.D. Thesis at Vrije Universiteit Amsterdam, 2007.
- [36] P.J. Mulders, J. Rodrigues, Phys. Rev. D 63 (2001) 094021, hep-ph/0009343.
- [37] S. Meissner, A. Metz, K. Goeke, Phys. Rev. D 76 (2007) 034002, hep-ph/0703176.